5章　問題解答

５－１

予習

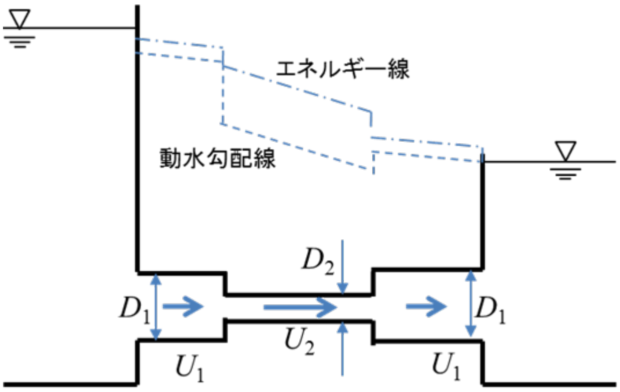
１． ①摩擦損失、②形状損失

２． 省略(参考ページ：第 4 章管路の流れ(1), pp.112～113)

３． 省略(参考ページ：第 4 章管路の流れ(1), pp.123～130)

演習問題　Ａ

5-1-A1



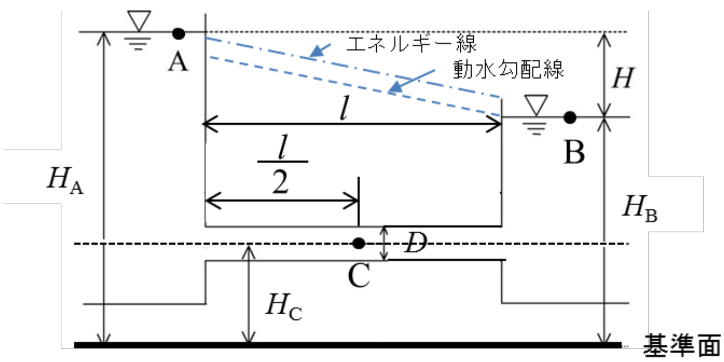
5-1-A2

(1)　$H = \left(K_e + f\dfrac{l}{D} + K_o\right)\dfrac{U^2}{2g}$　より

$$U = \sqrt{\frac{2gH}{\left(f_e + f\dfrac{l}{D} + f_o\right)}} = \sqrt{\frac{2\times 9.8\times(15-11)}{0.5+0.02\times\dfrac{200}{0.3}+1}} = 2.30\text{m/s}$$

$$Q = \frac{\pi D^2}{4}U = \frac{\pi\times 0.3^2}{4}\times 2.30 = 0.163\text{m}^3\text{/s}$$

(2)



(3)

$$\frac{p_\mathrm{C}}{\rho g} = H_\mathrm{A} - \left(K_e + f\frac{l/2}{D}\right)\frac{U^2}{2g} - H_C$$

$$= 15 - \left(0.5 + 0.02 \times \frac{100}{0.3}\right)\frac{2.30^2}{2 \times 9.8} - 5$$

$$= 8.07\mathrm{m}$$

5-1-A3

(1)

$$H = \left(f\frac{l}{D} + 1\right)\frac{U^2}{2g} \text{より、} U = \sqrt{\frac{2gH}{\left(f\frac{l}{D}+1\right)}}$$

$$Q = \frac{\pi D^2}{4}U = \frac{\pi D^2}{4}\sqrt{\frac{2gH}{\left(f\frac{l}{D}+1\right)}}$$

(2)

$$Q = \frac{\pi D^2}{4}\sqrt{\frac{2gH}{\left(f\frac{l}{D}+1\right)}} = \frac{\pi D^2}{4}\sqrt{\frac{2gHD}{fl + D}} \text{より、}$$

$$Q_* = \frac{\pi D^2}{4}\sqrt{\frac{2gH}{\left(f\frac{2l}{D}+1\right)}} = \frac{\pi D^2}{4}\sqrt{\frac{2gHD}{2fl + D}}$$

$$\frac{Q_*}{Q} = \sqrt{\frac{fl + D}{2fl + D}}$$

5-1-A4

① サイフォン　② 負圧　③ 大気圧

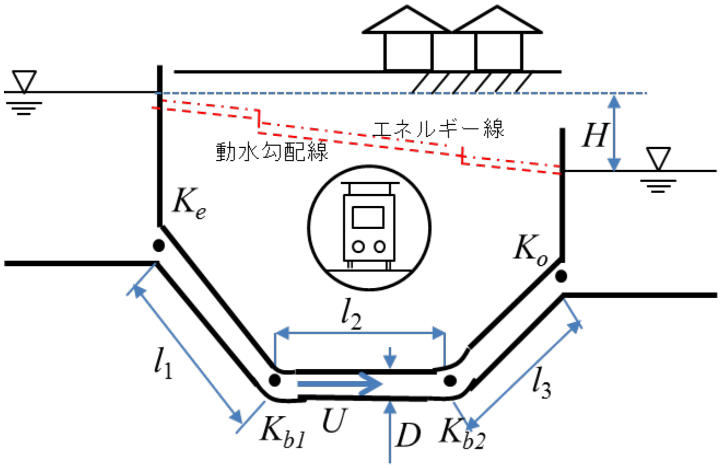
④ キャビテーション　⑤ −8

5-1-A5

上流側水路と下流側水路との間にベルヌーイの定理を適用すると、

$$H = (K_e + K_{b1} + K_{b2} + f\frac{l_1 + l_2 + l_3}{D} + K_o)\frac{U^2}{2g} \text{より、}$$

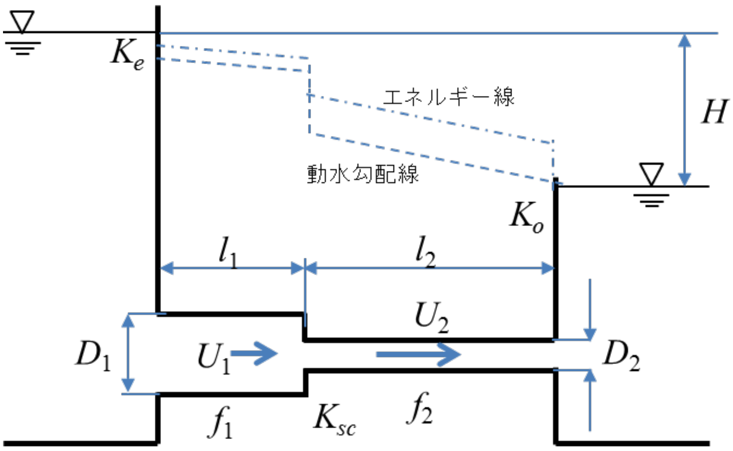
$$U = \sqrt{\frac{2gH}{(K_e + K_{b1} + K_{b2} + f\dfrac{l_1 + l_2 + l_3}{D} + K_o)}}$$



演習問題　B

5-1-B1

(1)



(2)

$$H = \left(K_e + f_1 \frac{l_1}{D_1}\right)\frac{U_1^{\ 2}}{2g} + \left(K_{sc} + f_2 \frac{l_2}{D_2} + K_o\right)\frac{U_2^{\ 2}}{2g}$$

(3)

$$U_1 \frac{\pi D_1^{\ 2}}{4} = U_2 \frac{\pi D_2^{\ 2}}{4} \text{ より、} U_1 = \frac{D_2^{\ 2}}{D_1^{\ 2}} U_2$$

(4)

$$H = \left\{\left(K_e + f_1 \frac{l_1}{D_1}\right)\frac{D_2^{\ 4}}{D_1^{\ 4}} + \left(K_{sc} + f_2 \frac{l_2}{D_2} + K_o\right)\right\}\frac{U_2^{\ 2}}{2g} \text{ より、}$$

$$U_2 = \sqrt{\frac{2gH}{\left(K_e + f_1\frac{l_1}{D_1}\right)\frac{D_2{}^4}{D_1{}^4} + \left(K_{sc} + f_2\frac{l_2}{D_2} + K_o\right)}}$$

$$= \sqrt{\frac{2\times 9.8\times 9}{\left(0.3+0.024\frac{25}{1.2}\right)\frac{0.8^4}{1.2^4} + \left(0.25+0.024\frac{40}{0.8}+1\right)}}$$

$=8.22\mathrm{m/s}$

$$Q = \frac{\pi D_2{}^2}{4}U_2 = \frac{\pi\times 0.8^2}{4}\times 8.22 = 4.13\mathrm{m^3/s}$$

5-1-B2

(1)

$$H = f\frac{l_1+l_2}{D}\frac{U^2}{2g} = 0.024\times\frac{25+75}{0.6}\frac{U^2}{2g} = \frac{2U^2}{g}$$

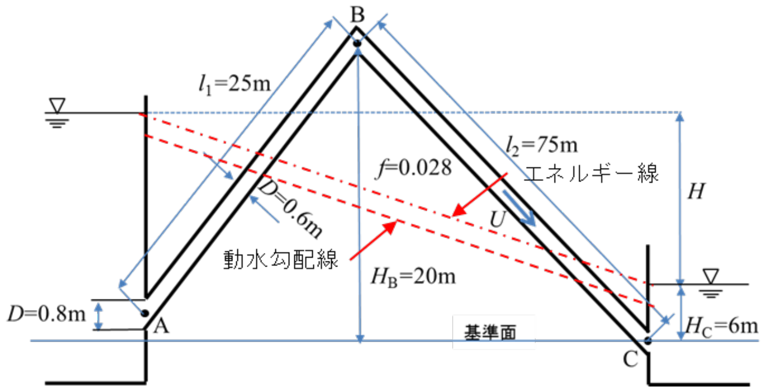
$$\frac{p_B}{\rho g} = H_C + H - \left(f\frac{l_1}{D}+1\right)\frac{U^2}{2g} - H_B \geq -8$$

$$H - \left(f\frac{l_1}{D}+1\right)\frac{U^2}{2g} \geq -8 - H_C + H_B = -8-6+20 = 6$$

$$H - \left(0.024\frac{25}{0.6}+1\right)\frac{U^2}{2g} = H - \frac{U^2}{g} = H - \frac{H}{2} = \frac{H}{2} \geq 6 \text{ より、}$$

$H \geq 12\,\mathrm{m}$

(2)



(3)

$$U=\sqrt{\frac{gH}{2}}=\sqrt{\frac{9.8\times 15}{2}}=8.57\ \mathrm{m/s}$$

(1)の第2式より，

$$\frac{p_B}{\rho g}=H_C+H-\left(f\frac{l_1}{D}+1\right)\frac{U^2}{2g}-H_B$$

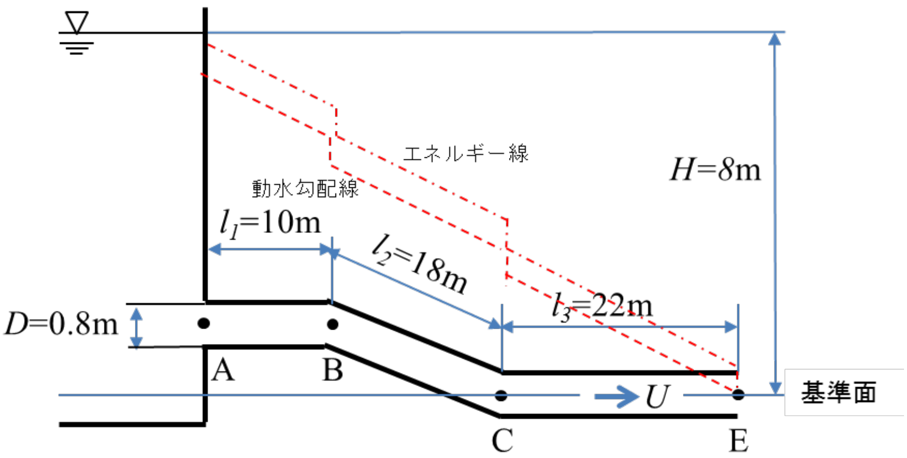
また,

$\dfrac{U^2}{2g}=\dfrac{H}{4}$ より

$$\frac{p_B}{\rho g}=6+15-\left(0.024\times\frac{25}{0.6}+1\right)\times\frac{15}{4}-20$$
$$=-6.5\mathrm{m}$$

5-1-B3

(1)



(2)

$$U=\sqrt{\frac{2gH}{K_e+2K_{be}+f\dfrac{l_1+l_2+l_3}{D}+1}}$$

$$=\sqrt{\frac{2\times 9.8\times 8}{0.56+2\times 0.3+0.028\dfrac{10+18+22}{0.8}+1}}$$

$=6.33\mathrm{m/s}$

$$Q=\frac{\pi D^2}{4}U=\frac{\pi\times0.8^2}{4}\times6.33=3.18\,\mathrm{m^3/s}$$

(3)

$$Q=\frac{\pi D^2}{4}\sqrt{\frac{2gH}{K_e+2K_{be}+f\dfrac{l_1+l_2+l_3}{D}+K_v+1}}$$

$$=\frac{\pi\times0.8^2}{4}\sqrt{\frac{2\times9.8\times8}{0.56+2\times0.3+0.028\dfrac{10+18+22}{0.8}+K_v+1}}$$

$$=\frac{6.294}{\sqrt{3.91+K_v}}=2.94$$

より、$K_v=\left(\dfrac{6.294}{2.94}\right)^2-3.91=0.67$

(4)

$$U=\sqrt{\frac{2gH}{f\dfrac{l_1+l_2+l_3}{D}+1}}=\sqrt{\frac{2\times9.8\times8}{0.028\dfrac{10+18+22}{0.8}+1}}=7.55\,\mathrm{m/s}$$

$$Q=\frac{\pi D^2}{4}U=\frac{\pi\times0.8^2}{4}\times7.55=3.80\,\mathrm{m^3/s}$$

$$\frac{p_C}{\rho g}=f\frac{l_3}{D}\frac{U^2}{2g}$$

$$=0.028\times\frac{22}{0.8}\times\frac{7.55^2}{2\times9.8}$$

$$=2.24\mathrm{m}$$

５－２

予習

１． ① 摩擦　② 形状　③ ２　④ 摩擦損失係数　⑤ 長さ　⑥ 直径

⑦ 急縮　⑧入口　⑨ 出口　⑩曲がり　⑪ $K_i\dfrac{U^2}{2g}$

２． 省略（参考ページ：第 4 章管路の流れ(1)，pp.118～119）

演習問題　Ａ

5-2-A1

$h_L=f\dfrac{l}{D}\dfrac{U_1^{\ 2}}{2g}=f\dfrac{2l}{D}\dfrac{U_2^{\ 2}}{2g}$ より、$U_2=\dfrac{\sqrt{2}}{2}U_1$

$$\frac{Q_2}{Q_1}=\frac{U_2}{U_1}=\frac{\sqrt{2}}{2}$$

5-2-A2

$$Q=Q_1+Q_2=\left(1+\frac{\sqrt{2}}{2}\right)Q_1=0.3 \text{ より、} Q_1=\frac{0.3}{1+\frac{\sqrt{2}}{2}}=0.176\ \mathrm{m^3/s}$$

$$Q_2=Q-Q_1=0.3-0.176=0.124\quad \mathrm{m^3/s}$$

5-2-A3

$$H=f\frac{l/2}{D}\frac{U_1{}^2}{2g}+f\frac{l/2}{D}\frac{(U_1/2)^2}{2g}=f\frac{5}{16}\frac{l}{D}\frac{U_1{}^2}{g}$$

$$U_1=\sqrt{\frac{16DgH}{5fl}}=\sqrt{\frac{16\times0.3\times9.8\times8.5}{5\times0.024\times800}}=2.04\ \mathrm{m/s}$$

$$Q=\frac{\pi D^2}{4}U_1=\frac{\pi\times0.3^2}{4}\times2.04=0.144\quad \mathrm{m^3/s}$$

また、点Aより上流の流量は、

$2U_1\dfrac{\pi D^2}{4}=2Q$　となるから、連続の式より穴から流出する流量は、

$2Q-Q=Q$ となる。

演習問題　B

5-2-B1

(1)

$$h_L=f_1\frac{l_1}{D_1}\frac{U_1{}^2}{2g}=f_2\frac{l_2}{D_2}\frac{U_2{}^2}{2g} \text{ より、} U_2=\sqrt{\frac{f_1}{f_2}\frac{l_1}{l_2}\frac{D_2}{D_1}}U_1$$

$$Q=\frac{\pi D_1{}^2}{4}U_1+\frac{\pi D_2{}^2}{4}U_2=\frac{\pi D_1{}^2}{4}U_1+\frac{\pi D_1{}^2}{4}\frac{D_2{}^2}{D_1{}^2}\sqrt{\frac{f_1}{f_2}\frac{l_1}{l_2}\frac{D_2}{D_1}}U_1$$

$$=\left(1+\frac{D_2{}^2}{D_1{}^2}\sqrt{\frac{f_1}{f_2}\frac{l_1}{l_2}\frac{D_2}{D_1}}\right)\frac{\pi D_1{}^2}{4}U_1=\left(1+\sqrt{\frac{f_1l_1D_2{}^5}{f_2l_2D_1{}^5}}\right)Q_1$$

$$Q_1=\frac{Q}{\left(1+\sqrt{\frac{f_1l_1D_2{}^5}{f_2l_2D_1{}^5}}\right)}$$

同様に、$Q_2 = \dfrac{Q}{\left(1+\sqrt{\dfrac{f_2 l_2 D_1{}^5}{f_1 l_1 D_2{}^5}}\right)}$

(2)

$$f_1 = \frac{124.5n^2}{D_1^{\frac{1}{3}}} = \frac{124.5\times 0.012^2}{0.2^{\frac{1}{3}}} = 0.0307$$

$$f_2 = \frac{124.5n^2}{D_2^{\frac{1}{3}}} = \frac{124.5\times 0.012^2}{0.1^{\frac{1}{3}}} = 0.0386$$

$$Q_1 = \frac{Q}{\left(1+\sqrt{\dfrac{f_1 l_1 D_2{}^5}{f_2 l_2 D_1{}^5}}\right)} = \frac{0.3}{\left(1+\sqrt{\dfrac{0.0307\times 120\times 0.1^5}{0.0386\times 30\times 0.2^5}}\right)} = 0.228\,\mathrm{m^3/s}$$

$$Q_2 = \frac{Q}{\left(1+\sqrt{\dfrac{f_2 l_2 D_1{}^5}{f_1 l_1 D_2{}^5}}\right)} = \frac{0.3}{\left(1+\sqrt{\dfrac{0.0386\times 30\times 0.2^5}{0.0307\times 120\times 0.1^5}}\right)} = 0.072\,\mathrm{m^3/s}$$

(3)

$$\frac{Q_1}{Q_2} = \frac{\left(1+\sqrt{\dfrac{f_1 l_2 D_1{}^5}{f_1 l_1 D_1{}^5}}\right)}{\left(1+\sqrt{\dfrac{f_1 l_1 D_1{}^5}{f_1 l_2 D_1{}^5}}\right)} = \frac{\left(1+\sqrt{\dfrac{l_2}{l_1}}\right)}{\left(1+\sqrt{\dfrac{l_1}{l_2}}\right)} = \frac{\left(1+\sqrt{\dfrac{30}{120}}\right)}{\left(1+\sqrt{\dfrac{120}{30}}\right)} = 0.5$$

5-2-B2

(1) $H = f\dfrac{l}{D}\dfrac{U_1{}^2}{2g} = \dfrac{fl}{2gD}U_1{}^2 = \dfrac{fl}{2gD}\left(\dfrac{Q_1}{\dfrac{\pi D^2}{4}}\right)^2 = \dfrac{8fl}{\pi^2 gD^5}Q_1{}^2$

(2) $H_\mathrm{B} - H = f\dfrac{l}{D}\dfrac{U_2{}^2}{2g} = \dfrac{fl}{2gD}U_2{}^2 = \dfrac{fl}{2gD}\left(\dfrac{Q_2}{\dfrac{\pi D^2}{4}}\right)^2 = \dfrac{8fl}{\pi^2 gD^5}Q_2{}^2$

(3) $H_\mathrm{C} - H = f\dfrac{l}{D}\dfrac{U_3{}^2}{2g} = \dfrac{fl}{2gD}U_3{}^2 = \dfrac{fl}{2gD}\left(\dfrac{Q_3}{\dfrac{\pi D^2}{4}}\right)^2 = \dfrac{8fl}{\pi^2 gD^5}Q_3{}^2$

(4) $Q_1 = Q_2 + Q_3$

(5) $H = \dfrac{8fl}{\pi^2 gD^5}\left(Q_2 + Q_3\right)^2$

$$H_\mathrm{B} = \frac{8fl}{\pi^2 gD^5}Q_2{}^2 + H = \frac{8fl}{\pi^2 gD^5}\left(2Q_2{}^2 + 2Q_2Q_3 + Q_3{}^2\right)$$

$H_\mathrm{C} = \dfrac{8fl}{\pi^2 gD^5}Q_3{}^2 + H = \dfrac{8fl}{\pi^2 gD^5}\left(Q_2{}^2 + 2Q_2Q_3 + 2Q_3{}^2\right)$　より、

$K_1 = \dfrac{8fl}{\pi^2 gD^5}$ とおくと、

$$H_\mathrm{C} - H_\mathrm{B} = K_1\left(Q_3{}^2 - Q_2{}^2\right)$$

ゆえに、$Q_3 = \sqrt{\dfrac{H_\mathrm{C} - H_\mathrm{B}}{K_1} + Q_2{}^2}$

(6) $\dfrac{H_\mathrm{C} - H_\mathrm{B}}{K_1} = K_2$ とおいて、H_B の式を書き換えると

$$H_\mathrm{B} = K_1\left(2Q_2{}^2 + 2Q_2Q_3 + Q_3{}^2\right) = K_1\left(2Q_2{}^2 + 2Q_2\sqrt{K_2 + Q_2{}^2} + K_2 + Q_2{}^2\right)$$

ここで、 $func(Q_2) = K_1\left(3Q_2{}^2 + 2Q_2\sqrt{K_2 + Q_2{}^2} + K_2\right) - H_\mathrm{B} = 0$

$func'(Q_2) = 6K_1Q_2 + 2K_1\sqrt{K_2 + Q_2{}^2} + \dfrac{2K_1Q_2{}^2}{\sqrt{K_2 + Q_2{}^2}}$ とおく。

これをニュートン・ラフソン法で解く。ただし、

$$K_1 = \frac{8fl}{\pi^2 gD^5} = \frac{8 \times 0.026 \times 120}{\pi^2 \times 9.8 \times 0.25^5} = 264.3$$

$$K_2 = \frac{H_\mathrm{C} - H_\mathrm{B}}{K_1} = \frac{18 - 12}{264.3} = 0.0227$$

計算手順は以下のとおり。

Q_2	$func(Q_2)$	$func''(Q_2)$	$Q_2 - \dfrac{func(Q_2)}{func'(Q_2)}$
0.200	52.179	533.868	0.102
0.102	12.133	288.732	0.060
0.060	2.044	193.099	0.050
0.050	0.119	170.798	0.049
0.049	0.000	169.363	0.049

$Q_2 = 0.049\ \mathrm{m^3/s}$

$$Q_3=\sqrt{K_2+Q_2{}^2}=\sqrt{0.0227+0.049^2}=0.158\,\mathrm{m^3/s}$$

$$Q_1=Q_2+Q_3=0.049+0.158=0.207\quad \mathrm{m^3/s}$$

5-2-B3

$$H_1=\frac{8fl_1}{\pi^2 gD^5}Q_1{}^2=\frac{8\times0.022\times300}{\pi^2\times9.8\times0.5^5}Q_1{}^2=17.47Q_1{}^2=K_1Q_1{}^2$$

$$H_2=\frac{8fl_2}{\pi^2 gD^5}Q_2{}^2=\frac{8\times0.022\times280}{\pi^2\times9.8\times0.5^5}Q_2{}^2=16.30Q_2{}^2=K_2Q_2{}^2$$

$$H_3=\frac{8fl_3}{\pi^2 gD^5}Q_2{}^2=\frac{8\times0.022\times450}{\pi^2\times9.8\times0.5^5}Q_3{}^2=26.20Q_3{}^2=K_3Q_3{}^2$$

ハーディークロス法を用いると、

修正回数	管路i	仮定流量Q_i	$K_iQ_i{}^2$	K_iQ_i	$Q-\frac{\sum K_iQ_i{}^2}{\sum 2K_iQ}$
I	1	2.2	84.5548	38.434	2.255
	2	0.8	-10.432	13.04	0.745
	3	1.8	-84.888	47.16	1.745
	Σ		-10.7652	98.634	
II	1	2.255	88.802	39.387	2.255
	2	0.745	-9.057	12.150	0.745
	3	1.745	-79.819	45.730	1.745
	Σ		-0.07	97.27	

したがって,

$Q_1=2.255\,\mathrm{m^3/s}$（A→B）

$Q_2=0.745\,\mathrm{m^3/s}$（A→C）

$Q_3=1.745\,\mathrm{m^3/s}$（C→B）

５－３

予習

１． 省略

２． ① ポンプ　　　　　　② 水力発電

３．

ポンプ動力は単位時間内に汲み上げる水の重量を水面から高所へ移動させるための仕事であるから，まず水を満たした水槽内の水の質量 m を求めると，

m=1×0.7×0.5×1000kg=350kg

となる。また、水槽の底から水を注入するとすれば，移動の高さは水面から水槽の重心までの距離となるから、

h=30＋0.5/2=30.25m

　したがって，仕事は mgh で与えられるので，T＝6 分間でこれだけの仕事を行ったとすると，動力は以下のようになる。

$P=mgh/T$=350×9.8×30.25/（6×60）=288W=0.288kW

演習問題　Ａ

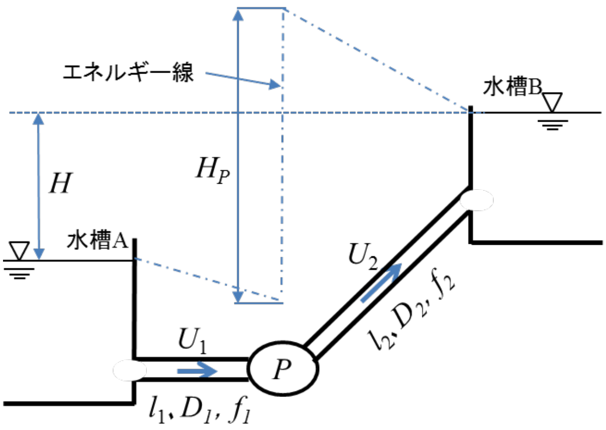
5-3-A1

① ポテンシャル　② 運動　③ 水車　④ 発電機　⑤ 水力発電
⑥ 機械的　⑦ 揚水　⑧ ポンプ

演習問題　Ｂ

5-3-B1

(1)



(2)

$$H_P = H + f_1 \frac{l_1}{D_1}\frac{U_1^{\,2}}{2g} + f_2 \frac{l_2}{D_2}\frac{U_2^{\,2}}{2g}$$

$$= H + f_1 \frac{l_1}{D_1}\frac{\left(4Q/\pi D_1^{\,2}\right)^2}{2g} + f_2 \frac{l_2}{D_2}\frac{\left(4Q/\pi D_2^{\,2}\right)^2}{2g}$$

$$= H + \left(\frac{8 f_1 l_1}{\pi^2 g D_1^{\,5}} + \frac{8 f_2 l_2}{\pi^2 g D_2^{\,5}}\right) Q^2$$

(3)

$$H_P = H + \left(\frac{8 f_1 l_1}{\pi^2 g D_1^{\,5}} + \frac{8 f_2 l_2}{\pi^2 g D_2^{\,5}}\right) Q^2$$

$$= 7 + \left(\frac{8 \times 0.022 \times 10}{\pi^2 \times 9.8 \times 0.5^5} + \frac{8 \times 0.024 \times 14}{\pi^2 \times 9.8 \times 0.4^5}\right) \times 0.4^2$$

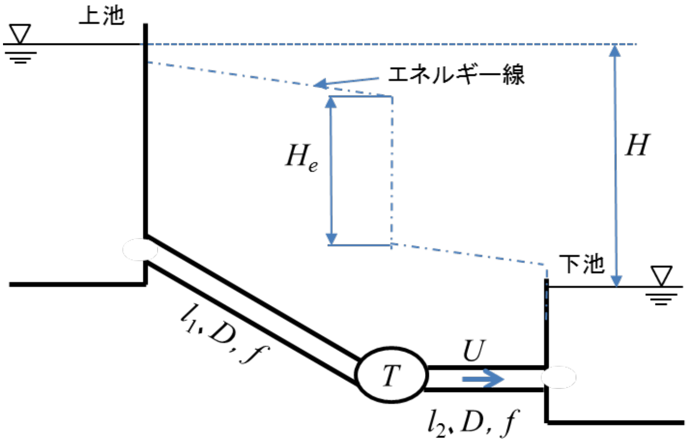
$$= 7.527\mathrm{m}$$

$$P_P = \frac{\rho g Q H_P}{\eta_P} = \frac{1000 \times 9.8 \times 0.4 \times 7.527}{0.74}$$

$= 39900$ W=39.9kW

5-3-B2

(1)



$$H = H_T + \left(f_e + f\frac{l_1 + l_2}{D} + 1 \right)\frac{U^2}{2g} == H_T + \frac{8Q^2}{\pi^2 g D^4}\left(f_e + f\frac{l_1 + l_2}{D} + 1 \right)$$

$$(2) = H_T + \frac{8 \times 2.8^2}{\pi^2 \times 9.8 \times 1.2^4}\left(0.6 + 0.022 \times \frac{500 + 200}{1.2} + 1 \right)$$

$$= H_T + 0.3127 \times 14.43 = 45$$

$$H_T = 45 - 4.512 = 40.49 \text{ m}$$

$$P_T = \eta_T \rho g Q H_P = 0.8 \times 1000 \times 9.8 \times 2.8 \times 40.49$$

=889000W=889kW