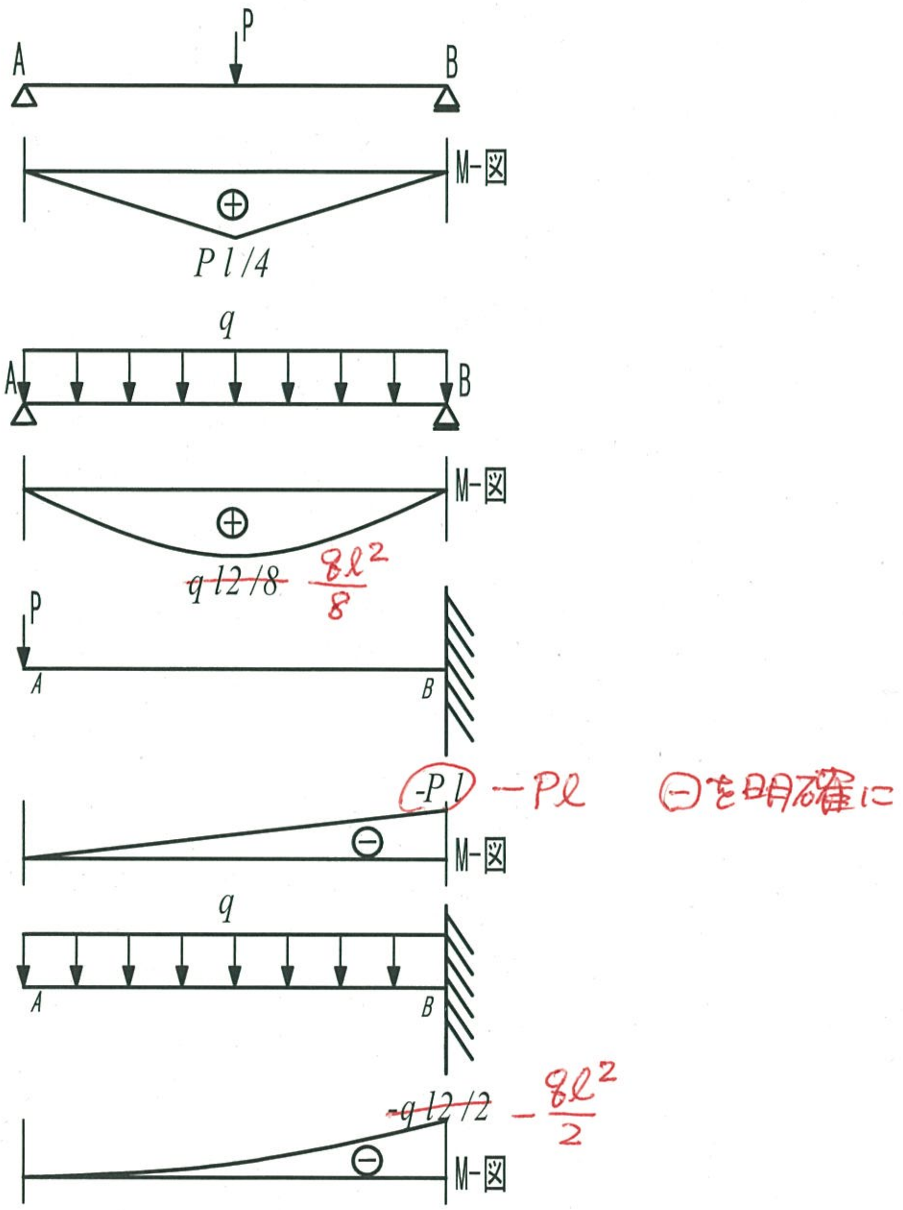
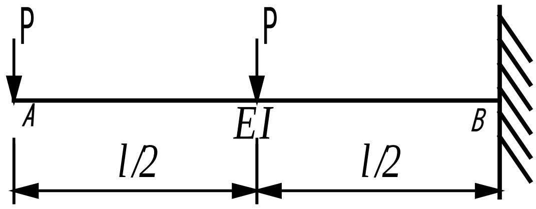
予習 授業の前にやっておこう!!

2. 右図のはりの曲げモーメント図を求めよ。

1. 集中荷重が載荷された単純ばり
2. 等分布荷重が載荷された単純ばり
3. 集中荷重が載荷された片持ちばり
4. 等分布荷重が載荷された片持ちばり



演習問題Ａ1　①たわみの微分方程式



片持ちばり(1)

［解答］

$0 \le x \le l/2 \quad \dfrac{d^2 v_1}{dx^2} = \dfrac{P}{EI} x$

$$\theta_1 = \frac{dv_1}{dx} = \frac{P}{2EI} x^2 + C_1 \qquad (1)$$

$$v_1 = \frac{P}{6EI} x^3 + C_1 x + C_2 \qquad (2)$$

$l/2 \le x \le l \quad \dfrac{d^2 v_2}{dx^2} = \dfrac{1}{EI}\left(2Px - \dfrac{Pl}{2}\right)$

$$\theta_2 = \frac{dv_2}{dx} = \frac{1}{EI}\left(Px^2 - \frac{Pl}{2}x\right) + D_1 \qquad (3)$$

$$v_2 = \frac{1}{EI}\left(\frac{Pl}{3}x^3 - \frac{Pl}{4}x^2\right) + D_1 x + D_2 \qquad (4)$$

固定端の境界条件： $x = l$ で　$\theta_2 = \dfrac{dv_2}{dx} = 0$　①，$v_2 = 0$　②

はりの連続条件： $x = l/2$ で $\theta_1 = \dfrac{dv_1}{dx} = \theta_2 = \dfrac{dv_2}{dx}$　③，$v_1 = v_2$　④

式(3)，式①より，$\theta_2 = \dfrac{dv_2}{dx} = \dfrac{1}{EI}\left(Pl^2 - \dfrac{Pl^2}{2}\right) + D_1 = 0 \qquad \therefore D_1 = -\dfrac{Pl^2}{2EI}$

式(4)，式②より，$v_2 = \dfrac{1}{EI}\left(\dfrac{Pl^3}{3} - \dfrac{Pl^3}{4}\right) - \dfrac{Pl^3}{2EI} + D_2 = 0 \qquad \therefore D_2 = \dfrac{5Pl^3}{12EI}$

式(1), (3)，式③より，$\dfrac{Pl^2}{8EI} + C_1 = \dfrac{1}{EI}\left(\dfrac{Pl^2}{4} - \dfrac{Pl^2}{4}\right) + \dfrac{Pl^2}{2EI} \qquad \therefore C_1 = -\dfrac{5Pl^2}{8EI}$

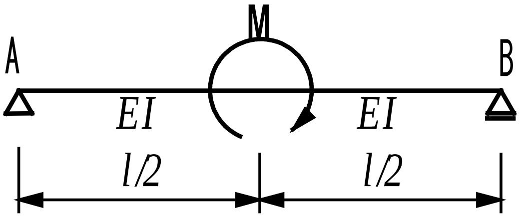
式(2), (4)，式④より，$\dfrac{Pl^3}{48EI} - \dfrac{5Pl^3}{16EI} + C_2 = \dfrac{1}{EI}\left(\dfrac{Pl^3}{24} - \dfrac{Pl^3}{16}\right) - \dfrac{Pl^3}{4EI} + \dfrac{5Pl^3}{12EI}$

$$\therefore C_2 = \frac{7Pl^3}{16EI}$$

$$\theta_1 = \frac{P}{2EI}x^2 - \frac{5Pl^2}{8EI} \qquad \theta_2 = \frac{P}{EI}x^2 - \frac{Pl}{2EI}x - \frac{Pl^2}{2EI}$$

$$v_1 = \frac{P}{6EI}x^3 - \frac{5Pl^2}{8EI}x + \frac{7Pl^3}{16EI} \qquad v_2 = \frac{P}{3EI}x^3 - \frac{Pl}{4EI}x^2 - \frac{Pl^2}{2EI}x + \frac{5Pl^3}{12EI}$$

演習問題Ａ1　②たわみの微分方程式



単純ばり(2)

［解答］

$$0 \le x \le l/2 \quad \frac{d^2v_1}{dx^2} = \frac{1}{EI}\left(\frac{M}{l}x\right)$$

$$\theta_1 = \frac{dv_1}{dx} = \frac{1}{EI}\left(\frac{M}{2l}x^2\right) + C_1 \qquad (1)$$

$$v_1 = \frac{1}{EI}\left(\frac{M}{6l}x^3\right) + C_1x + C_2 \qquad (2)$$

$$l/2 \le x \le l \quad \frac{d^2v_2}{dx^2} = \frac{1}{EI}\left(\frac{M}{l}x - M\right)$$

$$\theta_2 = \frac{dv_2}{dx} = \frac{1}{EI}\left(\frac{M}{2l}x^2 - Mx\right) + D_1 \qquad (3)$$

$$v_2 = \frac{1}{EI}\left(\frac{M}{6l}x^3 - \frac{M}{2}x^2\right) + D_1x + D_2 \qquad (4)$$

単純支持の境界条件： $x = 0$ で　$v_1 = 0$　①，$x = l$ で　$v_2 = 0$　②

はりの連続条件： $x = l/2$ で $\theta_1 = \frac{dv_1}{dx} = \theta_2 = \frac{dv_2}{dx}$　③，$v_1 = v_2$　④

式(2)，式①より， $v_1 = C_2 = 0$

式(4)，式②より， $v_2 = \frac{1}{EI}\left(\frac{Ml^2}{6} - \frac{Ml^2}{2}\right) + D_1 l + D_2 = 0 \quad \therefore D_1 l + D_2 = \frac{Ml^2}{3EI}$ (5)

式(1)，(3)，式③より， $\frac{Ml}{8EI} + C_1 = \frac{Ml}{8EI} - \frac{Ml}{2EI} + D_1 \quad \therefore C_1 - D_1 = -\frac{Ml}{2EI}$ (6)

式(2)，(4)，式④より， $\frac{Ml^2}{48EI} + C_1\frac{l}{2} = \frac{Ml^2}{48EI} - \frac{Ml^2}{8EI} + D_1\frac{l}{2} + D_2$

$$\therefore \left(C_1 - D_1\right)\frac{l}{2} - D_2 = -\frac{Ml^2}{8EI} \quad (7)$$

式(6)，(7)より， $-\frac{Ml^2}{8EI} - D_2 = -\frac{Ml^2}{8EI} \quad \therefore D_2 = -\frac{Ml^2}{8EI}$ (8)

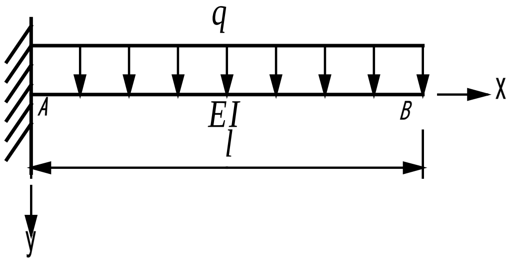
式(5)，(8)より， $D_1 l - \frac{Ml^2}{8EI} = \frac{Ml^2}{3EI} \quad \therefore D_1 = \frac{11Ml}{24EI}$ (9)

式(6)，(9)より， $C_1 - \frac{11Ml}{24EI} = -\frac{Ml}{2EI} \quad \therefore C_1 = -\frac{Ml}{24EI}$

$$\theta_1 = \frac{1}{EI}\left(\frac{M}{2l}x^2\right) - \frac{Ml}{24EI} \qquad \theta_2 = \frac{1}{EI}\left(\frac{M}{2l}x^2 - Mx\right) + \frac{11Ml}{24EI}$$

$$v_1 = \frac{1}{EI}\left(\frac{M}{6l}x^3\right) - \frac{Ml}{24EI}x \qquad v_2 = \frac{1}{EI}\left(\frac{M}{6l}x^3 - \frac{M}{2}x^2\right) + \frac{11Ml}{24EI}x - \frac{Ml^2}{8EI}$$

演習問題Ａ2　①たわみの微分方程式



片持ちばり(1)

［解答］

$$\frac{d^4v}{dx^4}=\frac{q}{EI} \qquad (1)$$

$$\frac{d^3v}{dx^3}=\frac{q}{EI}x+C_1 \qquad (2)$$

$$\frac{d^2v}{dx^2}=\frac{q}{2EI}x^2+C_1x+C_2 \qquad (3)$$

$$\theta=\frac{dv}{dx}=\frac{q}{6EI}x^3+\frac{C_1}{2}x^2+C_2x+C_3 \qquad (4)$$

$$v=\frac{q}{24EI}x^4+\frac{C_1}{6}x^3+\frac{C_2}{2}x^2+C_3x+C_4 \qquad (5)$$

固定端の境界条件： $x=0$ で　$\theta=\dfrac{dv}{dx}=0$　①，$v=0$　②

また，$x=l$ で　せん断力 $Q=0$　③，$x=l$ で　曲げモーメント $M=0$　④

式(4)，式①より，$\theta=C_3=0$

式(5)，式②より，$v=C_4=0$

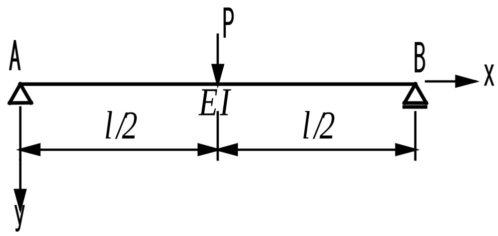
式(2)，式③より，$\dfrac{d^3v}{dx^3}=\dfrac{ql}{EI}+C_1=0$　　$\therefore C_1=-\dfrac{ql}{EI}$

式(3)，式④より，$\dfrac{d^2v}{dx^2}=\dfrac{ql^2}{2EI}-\dfrac{ql^2}{EI}+C_2=0$　　$\therefore C_2=\dfrac{ql^2}{2EI}$

$$\theta=\frac{dv}{dx}=\frac{q}{6EI}x^3-\frac{ql}{2EI}x^2+\frac{ql^2}{2EI}x$$

$$v = \frac{q}{24EI}x^4 - \frac{ql}{12EI}x^3 + \frac{ql^2}{4EI}x^2$$

演習問題Ａ2　②たわみの微分方程式



単純ばり(2)

［解答］

$0 \le x \le l/2$　$\frac{d^4 v_1}{dx^4} = 0$　(1)

$\frac{d^3 v_1}{dx^3} = C_1$　(2)

$\frac{d^2 v_1}{dx^2} = C_1 x + C_2$　(3)

$\theta_1 = \frac{dv_1}{dx} = \frac{C_1}{2}x^2 + C_2 x + C_3$　(4)

$v_1 = \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3 x + C_4$　(5)

単純支持の境界条件： $x = 0$ で $v_1 = 0$　①

はりの連続条件： $x = l/2$ で $\theta_1 = \frac{dv_1}{dx} = 0$　②

また， $x = 0$ で　せん断力 $Q = \frac{P}{2}$　③，曲げモーメント $M = 0$　④

式(2)，式③より， $\frac{d^3 v_1}{dx^3} = C_1 = -\frac{Q}{EI} = -\frac{P}{2EI}$

式(3)，式④より， $\frac{d^2 v_1}{dx^2} = C_2 = 0$

式(4)，式②より， $\theta_1 = \frac{dv_1}{dx} = -\frac{P}{4EI} \times \frac{l^2}{4} + C_3 = 0$　　$C_3 = \frac{Pl^2}{16EI}$

式(5)，式①より， $v_1 = C_4 = 0$

$$\theta_1 = \frac{dv_1}{dx} = -\frac{P}{4EI}x^2 + \frac{Pl^2}{16EI}$$

$$v_1 = -\frac{P}{12EI}x^3 + \frac{Pl^2}{16EI}x$$

$l/2 \le x \le l$ $\quad \dfrac{d^4 v_2}{dx^4} = 0$ (6)

$$\frac{d^3 v_2}{dx^3} = D_1 \qquad (7)$$

$$\frac{d^2 v_2}{dx^2} = D_1 x + D_2 \qquad (8)$$

$$\theta_2 = \frac{dv_2}{dx} = \frac{D_1}{2}x^2 + D_2 x + D_3 \qquad (9)$$

$$v_2 = \frac{D_1}{6}x^3 + \frac{D_2}{2}x^2 + D_3 x + D_4 \qquad (10)$$

単純支持の境界条件： $x = l$ で $v_2 = 0$ ⑤

はりの連続条件： $x = l/2$ で $\theta_2 = \dfrac{dv_2}{dx} = 0$ ⑥

また， $x = l$ で　せん断力 $Q = -\dfrac{P}{2}$ ⑦，曲げモーメント $M = 0$ ⑧

式(7)，式⑦より， $\dfrac{d^3 v_2}{dx^3} = D_1 = -\dfrac{Q}{EI} = \dfrac{P}{2EI}$

式(8)，式⑧より， $\dfrac{d^2 v_2}{dx^2} = \dfrac{Pl}{2EI} + D_2 = 0 \qquad D_2 = -\dfrac{Pl}{2EI}$

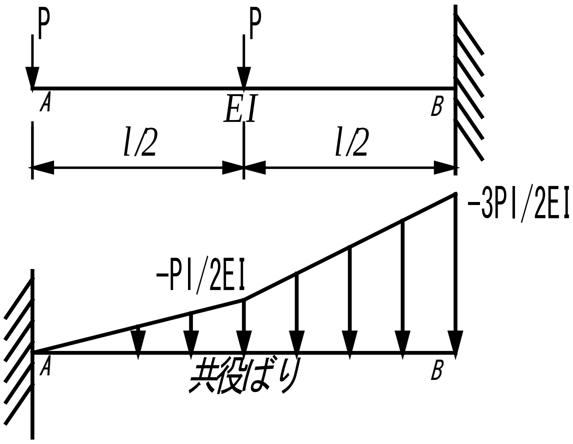
式(9)，式⑥より， $\theta_2 = \dfrac{dv_2}{dx} = \dfrac{P}{4EI} \times \dfrac{l^2}{4} - \dfrac{Pl}{2EI} \times \dfrac{l}{2} + D_3 = 0 \qquad D_3 = \dfrac{3Pl^2}{16EI}$

式(10)，式⑤より， $v_2 = \dfrac{Pl^3}{12EI} - \dfrac{Pl^3}{4EI} + \dfrac{3Pl^3}{16EI} + D_4 = 0 \qquad D_4 = -\dfrac{Pl^3}{48EI}$

$$\theta_2 = \frac{dy_2}{dx} = \frac{P}{4EI}x^2 - \frac{Pl}{2EI}x + \frac{3Pl^2}{16EI}$$

$$v_2 = \frac{P}{12EI}x^3 - \frac{Pl}{4EI}x^2 + \frac{3Pl^2}{16EI}x - \frac{Pl^3}{48EI}$$

演習問題Ａ3　弾性荷重法①



片持ちばり(1)－θ_Aおよびy_A

［解答］

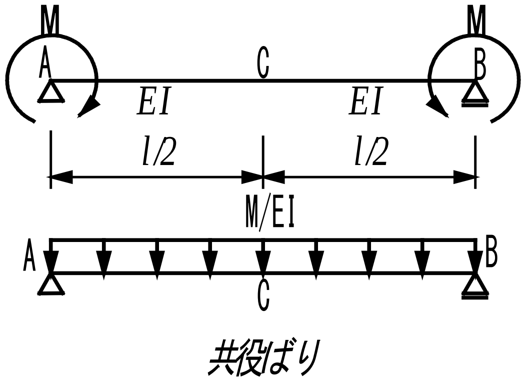
$$\sum V=0\ :\ V_A^*-\frac{1}{2}\times\left(\frac{-Pl}{2EI}\right)\times\frac{l}{2}-\frac{1}{2}\times\left(\frac{-Pl}{2EI}-\frac{3Pl}{2EI}\right)\times\frac{l}{2}=0$$

$$\theta_A=Q_A^*=V_A^*=-\frac{5Pl^2}{8EI}$$

$$\sum M_{(A)}=0\ :\ M_A^*+\left(\frac{-Pl^2}{8EI}\right)\times\left(\frac{l}{2}\times\frac{2}{3}\right)+\left(\frac{-Pl^2}{4EI}\right)\times\left(\frac{l}{2}+\frac{l}{2}\times\frac{2}{3}\right)+\left(\frac{-Pl^2}{4EI}\right)\times\left(\frac{l}{2}+\frac{l}{4}\right)=0$$

$$v_A=M_A^*=\frac{7Pl^3}{16EI}$$

演習問題Ａ3　弾性荷重法②



単純ばり(2)　－θ A および y_C

［解答］

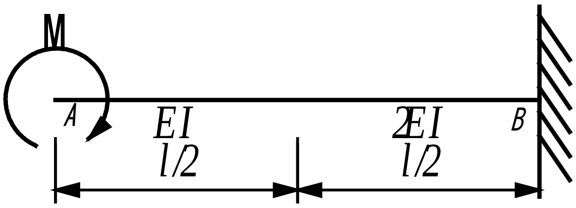
$$\sum M_{(\mathrm{B})}=0\ :\ V_A^* l-\frac{M}{EI}\times l\times\frac{l}{2}=0$$

$$\theta_A=Q_A^*=V_A^*=\frac{Ml}{2EI}$$

$$\sum M_{(\mathrm{C})}=0\ :\ -M_C^*+R_A^*\times\frac{l}{2}-\frac{M}{EI}\times\frac{l}{2}\times\frac{l}{4}=0$$

$$v_C=M_C^*=\frac{Ml^2}{4EI}-\frac{Ml^2}{8EI}=\frac{Ml^2}{8EI}$$

演習問題Ｂ1　①たわみの微分方程式



片持ちばり(1)

［解答］

$0 \le x \le l/2 \quad \dfrac{d^2v_1}{dx^2} = -\dfrac{M}{EI}$

$$\theta_1 = \frac{dv_1}{dx} = -\frac{M}{EI}x + C_1 \qquad (1)$$

$$v_1 = -\frac{M}{2EI}x^2 + C_1x + C_2 \qquad (2)$$

$l/2 \le x \le l \quad \dfrac{d^2v_2}{dx^2} = -\dfrac{M}{2EI}$

$$\theta_2 = \frac{dv_2}{dx} = -\frac{M}{2EI}x + D_1 \qquad (3)$$

$$v_2 = -\frac{M}{4EI}x^2 + D_1x + D_2 \qquad (4)$$

固定端の境界条件： $x = l$ で　$\theta_2 = \dfrac{dv_2}{dx} = 0$　①，$v_2 = 0$　②

はりの連続条件： $x = l/2$ で $\theta_1 = \dfrac{dv_1}{dx} = \theta_2 = \dfrac{dv_2}{dx}$　③，$v_1 = v_2$　④

式(3)，式①より， $\theta_2 = \dfrac{dv_2}{dx} = -\dfrac{Ml}{2EI} + D_1 = 0$　　$\therefore D_1 = \dfrac{Ml}{2EI}$

式(4)，式②より， $v_2 = -\dfrac{Ml^2}{EI} + \dfrac{Ml^2}{2EI} + D_2 = 0$　　$\therefore D_2 = -\dfrac{Ml^2}{4EI}$

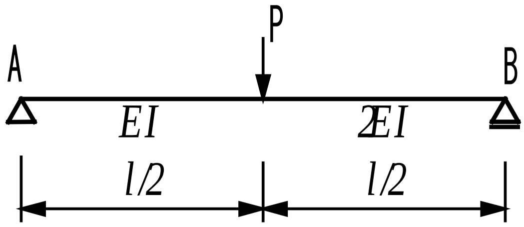
式(1)，(3)，式③より， $-\dfrac{Ml}{2EI} + C_1 = -\dfrac{Ml}{4EI} + \dfrac{Ml}{2EI}$　　$\therefore C_1 = \dfrac{3Ml}{4EI}$

式(2)，(4)，式④より， $-\dfrac{Ml^2}{8EI} + \dfrac{3Ml^2}{8EI} + C_2 = -\dfrac{Ml^2}{16EI} + \dfrac{Ml^2}{4EI} - \dfrac{Ml^2}{4EI}$　　$\therefore C_2 = -\dfrac{5Ml^2}{16EI}$

$\theta_1 = -\dfrac{M}{EI}x + \dfrac{3Ml}{4EI}$　　　$\theta_2 = -\dfrac{M}{2EI}x + \dfrac{Ml}{2EI}$

$v_1 = -\dfrac{M}{2EI}x^2 + \dfrac{3Ml}{4EI}x - \dfrac{5Ml^2}{16EI}$　　　$v_2 = -\dfrac{M}{4EI}x^2 + \dfrac{Ml}{2EI}x - \dfrac{Ml^2}{4EI}$

演習問題Ｂ１　②たわみの微分方程式



単純ばり(2)

［解答］

$0 \le x \le l/2 \quad \dfrac{d^2 v_1}{dx^2} = \dfrac{1}{EI}\left(-\dfrac{P}{2}x\right)$

$$\theta_1 = \frac{dv_1}{dx} = \frac{1}{EI}\left(-\frac{P}{4}x^2\right) + C_1 \qquad (1)$$

$$v_1 = \frac{1}{EI}\left(-\frac{P}{12}x^3\right) + C_1 x + C_2 \qquad (2)$$

$l/2 \le x \le l \quad \dfrac{d^2 v_2}{dx^2} = \dfrac{1}{2EI}\left(\dfrac{P}{2}x - \dfrac{Pl}{2}\right)$

$$\theta_2 = \frac{dv_2}{dx} = \frac{1}{2EI}\left(\frac{P}{4}x^2 - \frac{Pl}{2}x\right) + D_1 \qquad (3)$$

$$v_2 = \frac{1}{2EI}\left(\frac{P}{12}x^3 - \frac{Pl}{4}x^2\right) + D_1 x + D_2 \qquad (4)$$

単純支持の境界条件： $x = 0$ で　$v_1 = 0$　①，$x = l$ で　$v_2 = 0$　②

はりの連続条件： $x = l/2$ で $\theta_1 = \dfrac{dv_1}{dx} = \theta_2 = \dfrac{dv_2}{dx}$　③，$v_1 = v_2$　④

式(2)，式①より， $v_1 = C_2 = 0$

式(4)，式②より， $v_2 = \dfrac{1}{2EI}\left(\dfrac{Pl^3}{12} - \dfrac{Pl^3}{4}\right) + D_1 l + D_2 = 0 \quad \therefore D_1 l + D_2 = \dfrac{Pl^3}{12EI}$　(5)

式(1)，(3)，式③より， $-\dfrac{Pl^2}{16EI} + C_1 = \dfrac{1}{2EI}\left(\dfrac{Pl^2}{16} - \dfrac{Pl^2}{4}\right) + D_1$

$$\therefore C_1 - D_1 = -\frac{Pl^2}{32EI} \qquad (6)$$

式(2),(4)，式④より，$-\frac{Pl^3}{96EI} + C_1\frac{l}{2} = \frac{1}{2EI}\left(\frac{Pl^3}{96} - \frac{Pl^3}{32}\right) + D_1\frac{l}{2} + D_2$

$$\therefore \left(C_1 - D_1\right)\frac{l}{2} - D_2 = -\frac{Pl^3}{64EI} \qquad (7)$$

式(6),(7)より，$-\frac{Pl^3}{64EI} - D_2 = -\frac{Pl^3}{64EI}$　　$\therefore D_2 = 0$　(8)

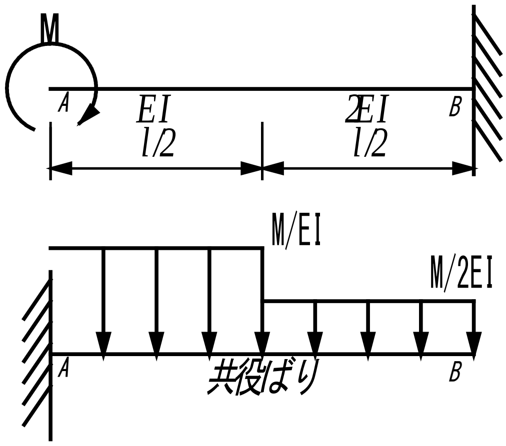
式(5),(8)より，$D_1 l = \frac{Pl^3}{12EI}$　　$\therefore D_1 = \frac{Pl^2}{12EI}$　(9)

式(6),(9)より，$C_1 - \frac{Pl^2}{12EI} = -\frac{Pl^3}{64EI}$　　$\therefore C_1 = \frac{5Pl^2}{96EI}$

$$\theta_1 = \frac{1}{EI}\left(-\frac{P}{4}x^2\right) + \frac{5Pl^2}{96EI} \qquad \theta_2 = \frac{1}{2EI}\left(\frac{P}{4}x^2 - \frac{Pl}{2}x\right) + \frac{Pl^2}{12EI}$$

$$v_1 = \frac{1}{EI}\left(-\frac{P}{12}x^3\right) - \frac{5Pl^2}{96EI}x \qquad v_2 = \frac{1}{2EI}\left(\frac{P}{12}x^3 - \frac{Pl}{4}x^2\right) + \frac{Pl^2}{12EI}x$$

演習問題Ｂ2　弾性荷重法①



片持ちばり(1) －θ_Aおよびy_A

［解答］

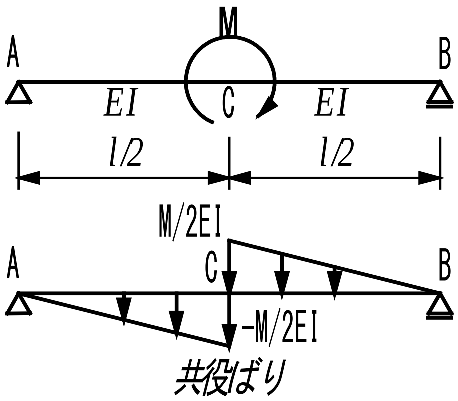
$$\sum V = 0 \;:\; V_A^* - \frac{M}{EI}\times\frac{l}{2} - \frac{M}{2EI}\times\frac{l}{2} = 0$$

$$\theta_A = Q_A^* = V_A^* = \frac{3Ml}{4EI}$$

$$\sum M_{(\mathrm{A})} = 0 \;:\; M_A^* + \frac{Ml}{2EI}\times\frac{l}{4} + \frac{Ml}{4EI}\times\left(\frac{l}{2}+\frac{l}{4}\right) = 0$$

$$v_A = M_A^* = -\frac{5Ml^2}{16EI}$$

演習問題Ｂ2　弾性荷重法②



単純ばり(2) －θ$_A$およびθ$_C$

［解答］

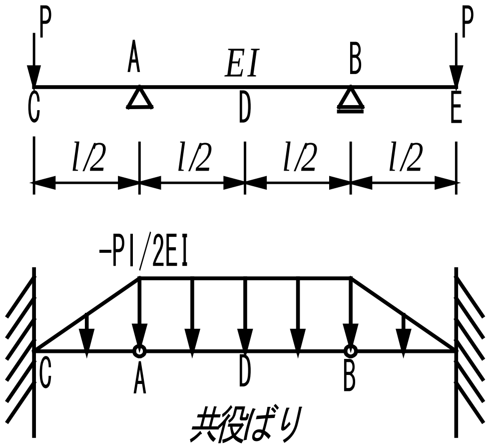
$$\sum M_{(B)} = 0 \ : \ V_A^* l - \frac{1}{2}\times\left(-\frac{M}{2EI}\right)\times\frac{l}{2}\times\left(\frac{l}{2}\times\frac{1}{3}+\frac{l}{2}\right)-\frac{1}{2}\times\frac{M}{2EI}\times\frac{l}{2}\times\left(\frac{l}{2}\times\frac{2}{3}\right)=0$$

$$\theta_A = Q_A^* = V_A^* = -\frac{Ml}{24EI}$$

$$\sum V = 0 \ : \ -Q_C^* + R_A^* - \frac{1}{2}\left(-\frac{M}{2EI}\right)\times\frac{l}{2} = 0$$

$$\theta_C = Q_C^* = -\frac{Ml}{24EI} + \frac{Ml}{8EI} = \frac{Ml}{12EI}$$

演習問題Ｂ2　弾性荷重法③



張出ばり(3) －θ C， y_C および y_D

［解答］

$$\sum V = 0 \ : \ V_C^* - \frac{1}{2}\times\left(-\frac{Pl}{2EI}\right)\times\frac{l}{2} - \left(-\frac{Pl}{2EI}\right)\times\frac{l}{2} = 0$$

$$\theta_C = Q_C^* = V_C^* - \frac{Pl^2}{8EI} - \frac{Pl^2}{4EI} = -\frac{3Pl^2}{8EI}$$

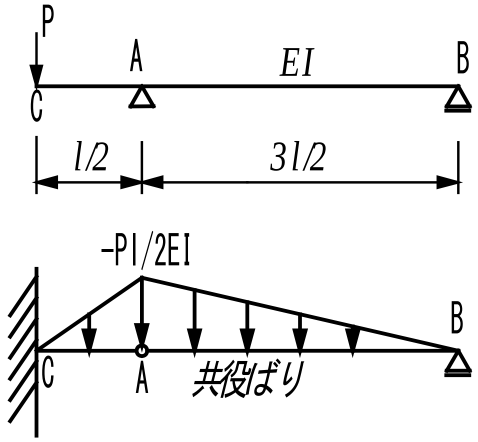
$$\sum M_{(\mathrm{C})} = 0 \ : \ M_C^* + \left(-\frac{Pl^2}{8EI}\right)\times\left(\frac{l}{2}\times\frac{2}{3}\right) + \left(-\frac{Pl^2}{4EI}\right)\times\frac{l}{2} = 0$$

$$v_C = M_C^* = \frac{Pl^3}{24EI} + \frac{Pl^3}{8EI} = \frac{Pl^3}{6EI}$$

$$\sum M_D = 0 \ : \ -M_D^* + \left(-\frac{Pl^2}{4EI}\right)\times\frac{l}{2} - \left(-\frac{Pl}{2EI}\right)\times\frac{l}{2}\times\frac{l}{4} = 0$$

$$v_D = M_D^* = -\frac{Pl^3}{8EI} + \frac{Pl^3}{16EI} = -\frac{Pl^3}{16EI}$$

演習問題Ｂ2　弾性荷重法④



張出ばり(4) －θ A， θ B， θ Cおよび y_C

［解答］

$$\sum V = 0 \ : \ V_A^* + V_B^* - \frac{1}{2} \times \left(-\frac{Pl}{2EI} \right) \times \frac{3l}{2} = 0$$

$$\sum M_{(B)} = 0 \ : \ V_A^* \times \frac{3l}{2} - \left(-\frac{3Pl^2}{8EI} \right) \times \left(\frac{3l}{2} \times \frac{2}{3} \right) = 0$$

$$V_A^* = -\frac{Pl^2}{4EI}, \quad V_B^* = \frac{Pl^2}{4EI} - \frac{3Pl^2}{8EI} = -\frac{Pl^2}{8EI}$$

$$\theta_A = Q_A^* = V_A^* = -\frac{Pl^2}{4EI}, \quad \theta_B = Q_B^* = -V_B^* = -\frac{Pl^2}{8EI}$$

$$\sum V = 0 \ : \ V_C^* - \frac{1}{2} \times \left(-\frac{Pl}{2EI} \right) \times \frac{l}{2} - \left(-\frac{Pl^2}{4EI} \right) = 0$$

$$\theta_C = Q_C^* = V_C^* = -\frac{Pl^2}{8EI} - \frac{Pl^2}{4EI} = -\frac{3Pl^2}{8EI}$$

$$\sum M_{(C)} = 0 \ : \ M_C^* + \left(-\frac{Pl^2}{8EI} \right) \times \left(\frac{l}{2} \times \frac{2}{3} \right) + \left(-\frac{Pl^2}{4EI} \right) \times \frac{l}{2} = 0$$

$$v_C = M_C^* = \frac{Pl^3}{24EI} + \frac{Pl^3}{8EI} = \frac{Pl^3}{6EI}$$